As we continue our journey through Calculus, there are certain skills that you learned this year which should be remembered/reviewed. Mastering these skills is crucial to your success not only in next year's class but also on the AP Calculus exam. For example, limits and derivatives are the two most important concepts one needs master because they are the fundamentals of calculus. Therefore, the rules of limits and differentiation should be mastered before diving deeper in to calculus.

This problem set is designed not just to help you review/relearn those topics but to re-familiarize yourself with AP style questions. I have added a link at the bottom of the page that could act as a resource if needed. You can also email me at pmcnally@chclc.org anytime this summer if you have questions.

I cannot stress enough how important it is to make sure you recall the concepts covered in this problem set. Take your time to relearn any topics you may have forgotten. If you do not feel confident with your answers to these questions or you need any guidance, please study your notes from our class, or reference the website below. Please do not wait until the last minute to complete the summer problem set. There are a lot of problems to solve and I want to make sure you take your time understanding each of them. At the same time, please do not finish the whole packet in the beginning of the summer. The point of this assignment is to refresh your memory of skills that are needed in the course and if you do all the work early, you may forget again before school starts.

I will collect the problem set on the first day of school. Since this is not a requirement but a recommendation, I will reward your hard work with 10% extra credit towards any graded assignment in the first marking period, including tests. Since I will not be able to monitor how you complete your packet, it is fine if you use a calculator. If you need to use a calculator, make sure your answers are given to 3 decimal places. However, keep in mind that half of the AP exam does not allow a calculator to be used, so don't rely too heavily on one. I expect the work to be done by you and you alone, you may not work with others to complete this problem set. Nor may you use the internet to search for answers.

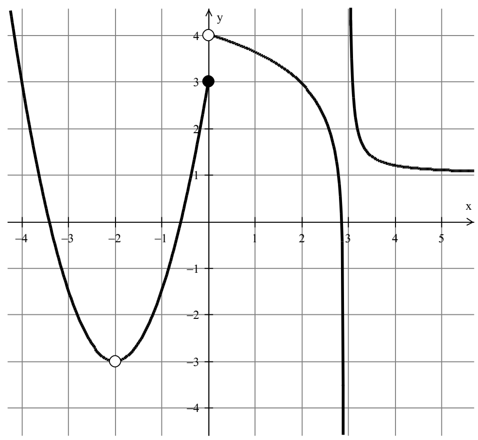
As I stated before, you may email me with any questions regarding the summer assignment. I look forward to working with you again in the fall. Good luck and have a great summer!

Helpful Website:

www.wolframalpha.com

Limits:

1. The graph of a function $y = f(x)$ is pictured below. For each portion of the question, give the value of each limit as a finite number, ∞, or $-\infty$, or state that the limit does not exist, if appropriate.



a) $\lim_{x\to 0^+} f(x) =$

b) $\lim_{x\to -2} f(x) =$

c) $\lim_{x\to \infty} f(x) =$

d) $\lim_{x\to 0} f(x) =$

e) $\lim_{x\to 2^+} f(f(x)) =$

f) $\lim_{x\to 3} f(x) =$

g) For what value(s) of x does f fail to be continuous? For each of these values of x, classify the discontinuity as being removable or nonremovable.

Compute each of the following limits.

2. $\lim_{x\to \pi/4} \dfrac{1-\tan x}{\cos 2x}$

3. $\lim_{x\to 0^+} \dfrac{\sin x - x}{x \sin x}$

Continuity and Differentiability:

4. Let $f(x)=\begin{cases}-6x^2+14x-8, x\le 1\\ \sin(2x-2), x>1\end{cases}$. Show that $f(x)$ is continuous and differentiable at $x=1$.

Derivatives:

5. If $y=5x\sqrt{x^2+5}$, then $\frac{dy}{dx}$ at $x=2$ is:

6. Let f be a differentiable function with $f(2)=3$ and $f'(2)=-5$, and let g be the function defined by $g(x)=x\cdot f(x)$. What is the equation for the line tangent to the graph of g at the point where $x=2$?

7. Suppose that f and g are twice differentiable functions having selected values given in the table below.

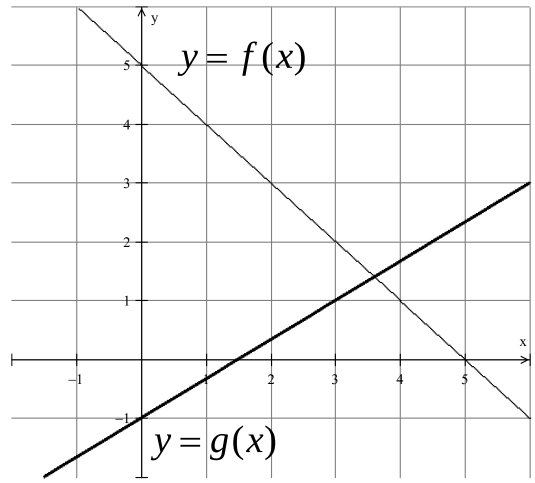
x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	4	2	7
2	8	6	–6	–4

If $h(x) = f(g(x))$, what is the value of $h'(x)$ at the point where $x = 1$?

8. Find the value of the limit: $\lim\limits_{h \to 0} \dfrac{\sqrt{\tan(2x+2h)} - \sqrt{\tan(2x)}}{h}$

9. A particle is moving along the $x-$axis such that its position at time $t \geq 0$ is described by the equation $x(t) = \dfrac{t-3}{2t+2}$. At the time when the velocity of the particle is equal to $\dfrac{1}{2}$, is the speed of the particle increasing or decreasing? Show the analysis that leads to your conclusion.

10. The graphs of f and g are shown below. If $p(x)=\dfrac{(f(x))^2}{g(x)}$, find the value of $p'(3)$.



11. The normal line to a function $f(x)$ at a point P is defined to be the line that passes through P and is perpendicular to the tangent line to f at that point. Find the equation of the normal line to $f(x)=x(1-2x)^3$ at the point $(1,-1)$.

12. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

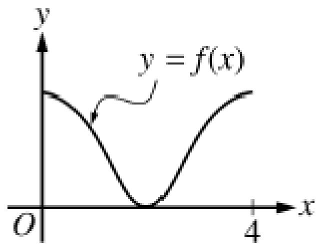
(a) Show that $\dfrac{dy}{dx} = \dfrac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

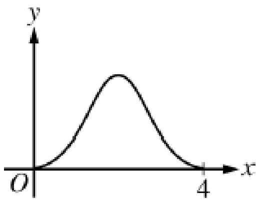
(c) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.

13. The function g is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute maximum value of g on the closed interval $[-2,1]$?

14.



The graph of $y = f(x)$ on the closed interval $[0, 4]$ is shown above. Which of the following could be the graph of $y = f'(x)$?

(A)


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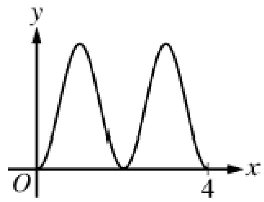
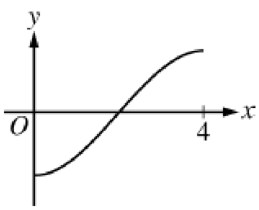


(C)


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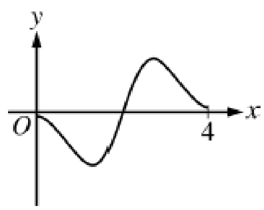

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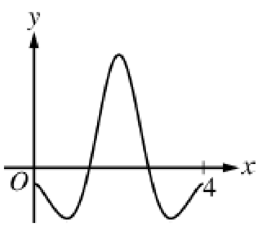
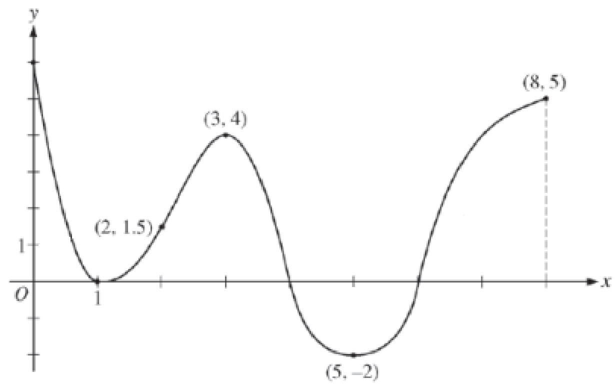


15. The figure below shows the graph of f', the derivative of a twice differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$ and $x = 5$, and the function f is defined for all real numbers.



Graph of f'

a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

b) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

c) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

d) Does the tangent line to the graph of $y = f(x)$ at the point where $x = 4$ lie above or below the curve near that point? Justify your response.

MVT and Rolle's Theorem:

16.

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

17.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	–20	–30	–20	–14	–10	0	10
$a(t)$ $\left(\text{ft}/\text{sec}^2\right)$	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(b) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.